BOOK REVIEWS

Plane and Solid Geometry. By W. W. Beman and D. E. Smith. Ginn & Co., Boston and London.

Mathematics has made rapid progress in the last hundred years, and the atmosphere of mathematical thought is as new and all-pervading as the material progress that has accompanied it. Elementary text-books, particularly geometries, seem, however, to carefully avoid the new ideas. The constant use of scissors and paste, and the manufacture of crude ideas along old-fashioned lines has gone on interminably. We shall certainly never have a geometry as worthy of our times as the geometry of Euclid was of the past, until a work is written that shows equally with Euclid's text, the results of logical insight, mathematical ability and knowledge, and laborious care in revision and adaptation to the needs of elementary students.

The text-book before me marks a step, and a long one, in the right direction, and has many of the elements that would fit it to become a representative work. No one can turn over its leaves without finding upon every page evidences of care in respect to logic and clearness, and of constant endeavor to place before the student ideas that will bring him into touch with modern geometrical methods. The essential feature of modern methods are to classify and reduce to general principles, thus strengthening the logical faculties and relieving the memory. In this respect, the authors have, for example, in their presentation of the principles of duality and of positive and negative relations and of continuity, enabled the student to grasp in one effort of the mind several related propositions, that stand as isolated theorems in geometries of ancient type. They have also presented the principles of symmetry, similarity and projection, in a way that leads the student naturally into the ideas and methods of homography and perspective. The explanations of the general methods for the attack and solution of geometrical propositions form an excellent feature of the book. There are many original propositions with which the student

may test his powers in this respect, and by which he may learn more of geometry than in any other way.

The book contains a number of propositions which is called modern geometry; although most of this geometry is nearly as old as Euclid, the modern part being the spirit of it. It seems to me that a modern text-book should not stop with a few such propositions but should go far enough to enable the student to handle the rule and compass in something like the modern way. Take, for example, the typical problem of the locus of the intersection of two tangents to a circle at the ends of a variable chord that passes through a fixed point. This leads up to the principle of pole and polar by which the exact character of the principle of duality may be established. This problem and its connection with harmonic division, form a class of properties of the circle of great practical importance. Supplement these with the projective properties of harmonic and anharmonic division, and the student has, under his control, a large number of theorems, that are useful in problems of construction and in the development of the properties of conic sections.

The logical arrangement of this geometry is particularly to be commended. It requires such wide knowledge and delicacy of insight, in separating legitimate reasoning from unfounded assumption, that the average text-book writer usually succeeds in filling the first part of his book with considerable trash in the way of argument. The book here reviewed shows no little care in avoiding the usual stumbling blocks. The authors have manifestly put considerable labor upon the question of logical arrangement and have placed it upon the most approved modern basis. The concepts that lie at the foundations of geometry have engaged the attention of the ablest mathematicians of modern and ancient times. The result has been, not only to vindicate Euclid in the most remarkable manner, but to establish new theories of plane and solid geometry that are also logically consistent, but which are so different from ordinary geometry as to be called non-Euclidean. Whether we live under the Euclidean régime or only approximately so, is a question of fact to be tested only by measurements of perfect accuracy.

While the general plan of development of the book is in accordance with modern results and investigations upon this subject, yet it is at fault in several minor points. These will doubtless be eliminated as the result of careful revision. *E. g.*, in the theory of limits it is

assumed that the variable always increases as it approaches its limit. The definition of continuity is not exploited sufficiently. As it is given [Theorems proved for one figure continue true for general figures, so long as the given conditions continue, . . .], it seems to justify the student who establishes a general proposition by the proof of a particular case. The force of the restriction, "so long as the given conditions continue," needs to be fully illustrated. The writer would object, also, to the definition of the straight line and plane both on account of their vagueness as definitions, and also on account of a certain lack of logical completeness. Thus, the definition of a straight line is "Through two points one straight line, and only one, can pass." This assumes, without so stating, that the straight line between two given points is also a straight line between any two of its intermediate points, else the definition is wholly ambiguous, *e. g.*, a line of quickest descent between two points would otherwise fulfill the condition, and so would almost any line between two points in respect to some quality.

In conclusion, the writer would say that, so far as he can judge by a careful examination, without the test of actual class-room work in teaching from the book, it promises to be a valuable text-book that is well entitled to the patronage of our schools.

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